ASYMMETRIC TIME CORRELATIONS IN TURBULENT SHEAR FLOWS

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Abstract Cross correlations between normal and downstream velocity fluctuations in a turbulent shear flow are shown to carry information about the non-normal amplification process. The creation of spanwise modulated streaks by downstream vortices implies an asymmetry in temporal correlation functions. We verify this in numerical simulations in shear flows with $Re_\lambda \approx 100$.

Keywords: Coherent structures, time correlations, liftup process

Coherent structures are an ubiquitous feature of turbulent shear flows (Townsend 1976, Holmes et al 1996). Various kinds of vortices, streaks or waves have been identified and considerable efforts have gone into identifying their dynamical origins and evolution. In boundary layers the non-normal amplification or lift-up effect (Landahl 1980, Boberg and Brosa 1988, Trefethen et al 1992, Grossmann 2000) is often an important source for coherent structures. Waleffe (1995, 1997) and Hamilton et al (1995) have discussed how lift-up and instabilities form a complete regeneration cycle that can explain sustained large scale fluctuations. The aim of our analysis is to find evidence for this process in statistical measures, in particular in temporal cross-correlation functions.

Following Waleffe (1995, 1997) and Hamilton et al (1995) the recylcing process has three steps: i) downstream vortices mix fluid in the normal direction and drive modulations in the downstream velocity, forming so-called streaks. ii) streaks undergo an instability to the formation of vortices pointing in the normal direction. iii) the mean shear profile now turns these vortices again in downstream direction, thus closing the loop. Of these processes the ones in step iii) and ii) are reasonably fast, whereas the one in i) is fairly slow, since it is connected with the lift-up and thus only linear in time. Evidence for this regeneration mechanism was found in various flows (Hamilton et al 1995, Waleffe 1995, 1997, Grossmann 2000). Within a dynamical system picture the regeneration

process can be connected to a periodic orbit, as in the case of Kawahare and Kida (2001). The complete application of this picture to turbulence is complicated not only by the presence of many more periodic orbits (as found by Schmiegel (unpublished) for a low-dimensional model), but also by the possibility of other spatial variabilities than just a periodic variation in spanwise and downstream directions. Therefore, in order to identify this process in fully developed turbulent flows other indicators have to be found.

The indicator for non-normal amplification that we focus on here is a temporal cross-correlation function (Eckhardt and Pandit 2002). Since the vortex drives the streak a cross-correlation between the vortex and the streak should be asymmetric in time: if the streak is probed after the vortex then there might be a correlation, if it is probed before then there should not be a correlation.

The origin of such correlations can be made clear in the context of a linear analysis around a laminar profile (Eckhardt and Pandit, 2002). let $\mathbf{u}_0 = Sy\mathbf{e}_x$ be the shear flow profile, let $\omega(t)$ the amplitude of a vortex and $s(t)$ be the amplitude of the streak, and assume that the nonlinear fluctuations can be modelled by white noise. The resulting linear stochastic model can be solved analytically and the cross-correlation between vortex and streak,

$$C_{\omega,s}(t) = \langle \omega(t')s(t'+t)\rangle_{t'} \,, \tag{1}$$

becomes

$$C_{\omega,s}(t) = \begin{cases} -Se^{\lambda t} & t<0 \\ -S(1+\lambda t)e^{-\lambda t} & t>0 \end{cases} . \tag{2}$$

As expected it is asymmetric. The sign of the correlation function follows from the sign of the velocity gradient: if the downstream velocity increases with y (positive S), then a positive velocity component will bring up slower fluid, hence make a negative change in the streak. Similarly, if it brings faster fluid down, it will make a positive contribution in downstream velocity, but with a negative vertical direction. So in both cases the cross correlation is negative. In addition, the correlation function is proportional to the shear rate S. The asymmetry follows from an additional term for positive times, so that the ratio

$$Q(t) = C(t)/C(-t) = 1 + \lambda t \tag{3}$$

is simply linear.

Clearly, this form of the correlation function is obtained under a series of assumptions, some of which are discussed in (Eckhardt and Pandit 2002), and further verification is required. In boundary layers the

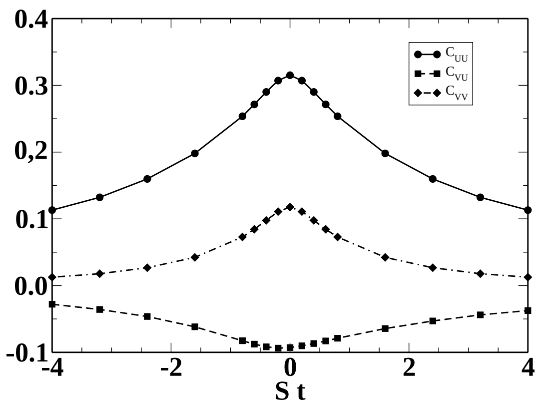

Figure 1. Temporal correlation functions in a turbulent shear flow. The downstream component is U, the normal component is V. Times are in dimensionless shear times, the Taylor Reynolds number is $Re_\lambda = 100$. The auto-correlation functions are symmetric, the cross-correlation function is negative and slightly asymmetric.

mean flow and Taylors frozen flow hypothesis are usually combined to translate temporal correlations into spatial correlations (e.g. Townsend 1976). This problem is avoided in a Lagrangian frame of reference or in a comoving frame without mean flow. Evidence for the asymmetry in a Lagrangian correlation function can be found in Fig. 1 of (Pope 2002), in a discussion of stochastic Lagrangian models. For Eulerian correlation functions we turn to our numerical simulations of a shear flow (Schumacher and Eckhardt 2000, Schumacher 2001).

The flow is bounded by parallel free-slip surfaces and driven by a steady volume force that maintains a linear shear profile (Schumacher and Eckhardt 2000). The statistical properties of the flow are in good agreement with other approximations to homogeneous shear flows (Schumacher 2001). What is important for our analysis is the fact that the mean downstream velocity vanishes: we can thus calculate correlation functions in a situation without mean flow (in the center) or with a slow mean flow (off-center, up to about half the distance to the surfaces).

The results from a simulation with $Re_\lambda \approx 100$ are shown in Fig. 1. The auto-correlation functions of the downstream (U) and normal (V) velocity components are symmetric in time. The cross-correlation function is negative and slightly asymmetric. In order to analyze the asymmetry further we show in Fig. 2 both a magnification of the central region and the ratio (3). The agreement with the linear prediction is satisfactory, considering the many non-linear processes in this fairly turbulent flow.

In summary, we have identified an asymmetry in temporal cross-correlation functions between downstream and normal velocity components in turbulent shear flows. The linearity of the asymmetry supports the connection to the liftup effect. For the analysis of experimental data the effects of a mean flow, of rigid boundaries and also of spatial

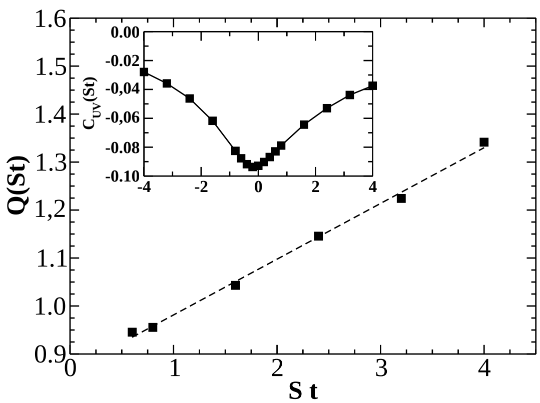

Figure 2. Asymmetry in the cross-correlation function for short times. The large frame shows the ratio of the cross-correlation function for positive and negative times, $Q(t) = C(t)/C(-t)$, where a linear behaviour is detected. The inset shows a magnification of the cross-correlation function of Fig. 1 near the center.

inhomogeneities have to be investigated. We have evidence that the effect is strongest near the boundary and decreases as one moves into the turbulent volume. This would be consistent with the decrease of the mean shear gradient and would indicate that non-normal amplification becomes less important further away from the boundaries.

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